2023

STATISTICS

(Honours Core)

Paper : STA-HC-3016

(Sampling Distributions)

Full Marks : 60

Time : Three hours

*The figures in the margin indicate
full marks for the questions.*

1.　Answer the following questions as directed :

$$1 \times 7 = 7$$

(a)　The standard error of the sampling distribution of the statistic $(\bar{x})$ is

(Choose the correct option)

(i)　$\sigma^2 \sqrt{2/n}$

(ii)　$\sigma/\sqrt{n}$

(iii)　$\sqrt{\sigma^2/2n}$

(iv)　None of the above

(b) The cumulative distribution function of the largest order statistic $X_{(n)}$ is given by _____ . *(Fill in the blank)*

(c) For large n if $X \sim N(nP, nPQ)$, then

$$Z = \frac{X - nP}{\sqrt{nPQ}} \text{ follows}$$

(Choose the correct option)

(i) $N\left(0, \sigma^2\right)$

(ii) $N\left(\mu, \sigma^2\right)$

(iii) $N(0, 1)$

(iv) $N(\mu, \sigma)$

(d) If $X_i, i = 1, 2, ...n$ are n independent normal variates with mean (μ_i) and SD (σ_i), then $\displaystyle\sum_{i=1}^{n} \left(\frac{X_i - \mu}{\sigma}\right)^2$ is a chi-square variate with n d.f.

*(Write true **or** false)*

(e) The probability of type I error is called
_____ . *(Fill in the blank)*

(f) State the pdf of Fisher's *t*-distribution.

(g) Equality of two population variances can be tested by
(Choose the correct option)

    (i) *t*-test

    (ii) *F*-test

    (iii) Both *(i)* and *(ii)*

    (iv) None of the above

2. Answer the following questions :   2×4=8

(a) Derive the cumulative distribution function of $X_{(1)}$.

(b) Write *any two* applications of chi-square statistic.

(c) Explain one tailed and two tailed tests.

(d) Write the assumptions for students *t*-test.

3. Answer *any three* questions from the following :   5×3=15

   (a) Explain in brief the test used for testing the difference between two proportions for large samples.

   (b) Find the joint distribution of rth and s^{th} order statistics $(r < s)$ in taking random sample from a continuous distribution.

   (c) Derive cumulant generating function (c.g.f.) of chi-square distribution. Also find its mean and variance using c.g.f.

   (d) Define *F* statistic. Write down the p.d.f. of Snedecor's *F* distribution. Derive the mode of *F* distribution.   1+1+3=5

(e) In $F(n_1, n_2)$ distribution and if $n_2 \to \infty$, then prove that $n_1 F$ follows chi-square distribution with n_1 d.f.

Answer **either** 4. (a) **or** 4. (b) :

4. (a) Obtain the distribution of sample median in case of order statistics. 10

(b) (i) Let X_1 and X_2 be two independent normal variates with the same normal distribution $N(\mu, \sigma^2)$. Obtain the distribution of

$$Y = \frac{X_1 + X_2 - 2\mu}{\sqrt{|X_1 - X_2|^2}}.$$ 5

(ii) If X is t-distributed with K degrees of freedom, show that $\dfrac{1}{1 + (X^2/K)}$ has a beta distribution. 5

Contd.

*Answer **either** 5. (a) **or** 5. (b) :*

5. (a) If X_1 and X_2 are two independent chi-square variate with n_1 and n_2 d.f. respectively, then show that X_1/X_2 is a $\beta_2(n_1/2, n_2/2)$ variate. 10

   (b) (i) Describe the steps in detail for testing a statistical hypothesis. 5

   (ii) For t-distribution with n d.f., derive the mean deviation about mean. 5

*Answer **either** 6. (a) **or** 6. (b) :*

6. (a) (i) Derive the probability density function of student's t. 7

   (ii) Comment on the graph of t-distribution. 3

(b) Write three applications of F distribution.

Let X_1 and X_2 be a random sample of size 2 from $N(0, 1)$ and Y_1 and Y_2 be a random sample of size 2 from $N(1, 1)$ and let Y_i's be independent of X_i's. Find the distribution of the following :

$$3+7=10$$

(i) $\dfrac{(X_1 + X_2)^2}{(X_2 - X_1)^2}$

(ii) $\dfrac{(Y_1 + Y_2 - 2)^2}{(X_2 - X_1)^2}$