2026

STATISTICS

(Major)

Paper : STA0600204

(Bivariate/Multivariate Analysis, Stochastic Process and Computer Programming)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions.

1. Answer the following as directed : 1×8=8

   (a) Write down the probability density function of the bivariate normal distribution.

   (b) An irreducible Markov chain contains

   (i) one closed set

   (ii) two closed sets

   (iii) three closed sets

   (iv) All of the above

   (Choose the correct option)

(c) Use of more than one main( ) function in every C program is illegal.

(State True or False)

(d) The characteristic function of the multivariate normal distribution having mean vector $\underset{\sim}{O}$ and variance-covariance matrix I, is ______.

(Fill in the blank)

(e) State any one property of transition probability matrix.

(f) Hotelling T^2 can be regarded as multivariate analog of the square of univariate t.

(State True or False)

(g) Every program segment in C language must end with a ______.

(Fill in the blank)

(h) State the moment generating function of the bivariate normal distribution.

2. Answer any *six* of the following questions : $2\times6=12$

(a) Find cov $(A\underset{\sim}{X}, B\underset{\sim}{Y})$, where A and B are matrices of constant elements.

(b) Write down the logical operators available in C language.

(c) Define Markov chain with an example.

(d) If $(X, Y) \sim \text{BVN}(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$, then state the marginal density functions of X and Y.

(e) Write down the operator precedence for arithmetic operators in C language.

(f) Write briefly on the specifications of Stochastic process.

(g) Let $\underset{\sim}{X} \sim N(\underset{\sim}{\mu}, \Sigma)$, where

$$\Sigma = \begin{pmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{pmatrix}, \quad \underset{\sim}{X} = (X_1 \ X_2 \ X_3)'$$

Are (X_1, X_2) and X_3 independent? Justify your answer.

(h) What is header file? Explain the need of the header file

include <stdio.h> 1+1=2

(i) State any two postulates for Poisson process.

(j) Define the following states of Markov chain :

Absorbing state, Persistent state

3. Answer any *four* of the following questions : 5×4=20

(a) If $\underset{\sim}{X} \sim N_p(\underset{\sim}{\mu}, \Sigma)$, then obtain the distribution of $\underset{\sim}{Z} = D\underset{\sim}{X}$ where D is a $p \times p$ non-singular matrix of constant elements.

(b) Define Poisson process. Prove the additive property of Poisson process. 1+4=5

(c) Describe the nested 'IF-ELSE' statement in C language with the help of a flowchart.

(d) Discuss the applications of Hotelling T^2 based on two samples.

(e) Given the following transition probability matrix :

$$P = \begin{pmatrix} q & p \\ p & q \end{pmatrix}, \text{ where } p+q=1$$

Find the probability of transition from state 0 to 1 in four steps.

(f) If $(X, Y) \sim \text{BVN}(0, 0, 1, 1, \rho)$, then show that $X+Y$ and $X-Y$ are independently distributed.

(g) Describe how to 'jump out' of a loop in C language.

(h) Suppose that the probability of a dry day (state 0) following a rainy day (state 1) is $\frac{1}{3}$ and the probability of a rainy day following a dry day is $\frac{1}{2}$. Given that May 1 is a dry day, find the probability that May 3 is also a dry day.

4. Answer any *two* of the following questions : 10×2=20

(a) Derive the p.d.f. of bivariate normal distribution as a particular case of multivariate normal distribution.

(b) (i) Write a detailed note on applications of Stochastic process. 5

(ii) If $\{N(t)\}$ is a Poisson process and $s < t$, then prove that

$$P\{N(s) = k / N(t) = n\} = \binom{n}{k} (s/t)^k (1 - s/t)^{n-k}$$ 5

(c) (i) Prove that Hotelling T^2 is invariant under a non-singular linear transformation. 6

(ii) For a bivariate normal distribution

$$f(x, y) = k \exp\left[-\frac{2}{3}(x^2 - xy + y^2 - 3x + 3y + 3)\right]$$

find the value of the constant k. 4

(d) (i) Write a detailed note on arithmetic operators available in C language. 4

(ii) Write a C program to determine the correlation coefficient for a bivariate distribution. 6

(e) (i) Write a short note on Markov chain as graphs. 4

(ii) Define matrix of transition probabilities. Let $\{X_t, t \geq 0\}$ be a Markov chain with three states 0, 1 and 2 with transition matrix

$$\begin{pmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{pmatrix}$$

and the initial distribution

$$P\{X_0 = i\} = \frac{1}{3}; \quad i = 0, 1, 2$$

Find—

$P(X_1 = 1 / X_0 = 2)$

$P(X_2 = 2, X_1 = 1 / X_0 = 2)$

and $P(X_2 = 2, X_1 = 1, X_0 = 2)$

1+1+2+2=6

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