Total number of printed pages–8

1 (Sem–4/FYUGP) STA 42 MJ

2026

STATISTICS

(Major)

Paper : STA4400204MJ

(Mathematical Methods)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : 1×8=8

(a) For positive integers n and m

(i) $\Delta^n 0^n = n!$ and $\Delta^m 0^n = 0, n > m$

(ii) $\Delta^n 0^n = 0$ and $\Delta^m 0^n = 0, n < m$

(iii) $\Delta^n 0^n = n!$ and $\Delta^m 0^n = 0, n < m$

(iv) None of the above

(Choose the correct option)

(b) A necessary condition for convergence of an infinite series ΣU_n is that $\lim_{n\to\infty} U_n =$ ________. *(Fill in the blank)*

(c) According to L Hospital's rule for indeterminate form %, under certain conditions imposed upon the function $f(x)$ and $g(x)$, if $\lim_{x\to a} \frac{f'(x)}{g'(x)}$ exists, then the value of $\lim_{x\to a} \frac{f(x)}{g(x)}$ is

(i) l

(ii) 0

(iii) $\lim_{x\to a} \frac{f'(x)}{g'(x)}$

(iv) $\lim_{r\to a+0} \frac{F(x)}{G(x)}$

(Choose the correct option)

(d) Define degree of a differential equation.

(e) Which of the following is not correct?

(i) $\mu = \frac{1}{2}\left[E^{1/2} + E^{-1/2}\right]$

(ii) $\mu^2 = 1 + \frac{1}{4}\delta^2$

(iii) $\mu\delta = \Delta + \nabla$

(iv) $E^{1/2} = \frac{1}{2}\delta + \mu$

(Choose the correct option)

(f) If x is an integer greater than $n-1$, then $x^{(n)} = \frac{x!}{(x-n)!}$.

*(State True **or** False)*

(g) Define Gamma function.

(h) For what purpose, regula falsi method is used?

(i) Numerical integration

(ii) Determination of the roots of a polynomial

(iii) Solution for differential equation

(iv) None of the above

(Choose the correct option)

2. Answer ***any six*** questions from the following : $2\times6=12$

(a) Define General and Particular Solutions of a differential equation.

(b) Solve the difference equation

$$U_{x+1} - aU_x = 0, a \neq 1.$$

(c) Obtain the function whose first difference is $9x^2 + 11x + 5$.

(d) Prove that $E \equiv e^{hD}$.
(Symbols have their usual meanings)

(e) Solve the following differential equation :

$$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$$

(f) Show that the series $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \ldots$ is not convergent.

(g) State Raabe's test for convergence of a series.

(h) Evaluate $\lim_{x \to 0} \frac{xe^x - \log(1+x)}{x^2}$

(i) If $x = r\cos\theta$ and $y = r\sin\theta$, find the Jacobian of the transformations of variables (x, y) to (r, θ).

(j) Find the value of $\beta(1/2, 1/2)$.

3. Answer ***any four*** questions from the following : $5\times4=20$

(a) Express the function

$f(x) = 2x^3 - 3x^2 + 3x - 10$ in a factorial notation and hence show that

$4^3 f(x) = 12$.

(b) If $x = u - v + w$

$$y = u^2 - v^2 - w^2$$

$$z = u^3 + v$$

then evaluate the Jacobian $\frac{\partial(x, y, z)}{\partial(u, v, w)}$

(c) Prove that $\beta(m, n) = \beta(n, m)$.

(d) Define positive term series. Show that the positive term geometric series $1 + r + r^2 + \ldots$ converges for $r < 1$.

(e) Show that $\Delta^n 0^m = n\left(\Delta^{n-1} 0^{m-1} + \Delta^n 0^{m-1}\right)$ where n and m are positive integer.

(f) Find the maximum and minimum values of the function

$$f(x) = 8x^5 - 15x^4 + 10x^2$$

(g) Solve the difference equation

$$u_{x+2} - 4u_{x+1} + 4u_x = 2^x$$

(h) Solve the differential equation

$$x\frac{dy}{dx} + 2y = x^2 \log x$$

4. Answer ***any two*** questions from the following : 10×2=20

(a) (i) State and prove Bessel's central difference formula. 5

(ii) If third differences are constant, prove that

$$y_{x+1/2} = \frac{1}{2}(y_x + y_{x+1}) - \frac{1}{16}\left(\Delta^2 y_{x-1} + \Delta^2 y_x\right)$$

5

(b) (i) Discuss the convergence of Regula Falsi method.

(ii) Prove that $\Gamma(1/4)\,\Gamma(3/4) = \sqrt{2}\pi$.

(c) (i) State the necessary and sufficient condition for the differential equation $Mdx + Ndy = 0$ to be exact. 1

(ii) Solve the differential equation $\left(x^2 + y^2 + x\right)dx + xy\,dy = 0$. 5

(iii) Obtain the differential equation of the family of curves $y = e^x(A\cos x + B\sin x)$. 4

(d) (i) Write a note on Lagrange's undetermined multipliers. 5

(ii) Show that if $2x + 3y + 4z = a$, the maximum value of $x^2y^3z^4$ is $\left(\frac{a}{9}\right)^9$. 5

(e) (i) Show that

$$\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\,dx = \beta(m, n)$$

for $m, n > 0$. 5

(ii) If a function $f(x)$ has maximum or minimum at a point $x = a$ within its domain, then show that $f'(a) = 0$. 5