Total number of printed pages–7

1 (Sem–4/FYUGP) STA 43 MJ

2026

STATISTICS

(Major)

Paper : STA4400304MJ

(Linear Algebra and System of Equations)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : $1 \times 8 = 8$

(a) State the condition that a matrix A has to satisfy to be an orthogonal matrix.

(b) A necessary and sufficient condition for a matrix A to be symmetric is that ______ and ______ are equal. *(Fill in the blanks)*

(c) If any two rows (or columns) of a determinant are interchanged, the value of the determinant is multiplied by ______. *(Fill in the blank)*

(d) If α, β, γ are the roots of the equation $x^3 - 5x^2 - 16x + 80 = 0$, then the product of the root is

(i) 5

(ii) -16

(iii) -80

(iv) None of the above

(Choose the correct option)

(e) The rank of a unit matrix of order n is

(i) 1

(ii) n

(iii) $n-1$

(iv) None of the above

(Choose the correct option)

(f) If the matrix is of order n, then the degree of characteristic equation $|A - XI| = 0$ is n. *(State True **or** False)*

(g) Given that for a (5×5) matrix A and $|A| = 59$, find $|3A|$.

(h) If $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is an eigenvector of $\begin{bmatrix} 1 & -n \\ -3 & 2n \end{bmatrix}$, then n is

(i) 1

(ii) -2

(iii) 3

(iv) 4

2. Answer ***any six*** questions from the following : $2 \times 6 = 12$

(a) Show that every orthogonal matrix is non-regular.

(b) Show that the vectors $x_1 = (1, 2, 4)$, $x_2 = (3, 6, 12)$ are linearly dependent.

(c) Find the rank of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

(d) Given for a (3×3) matrices $|adj\ A| = 20$, find $|A|$.

(e) Show that λ is a characteristic root of the matrix A if and only if there exists a non-zero vector X such that $AX = \lambda X$.

(f) Determine the eigenvalues of the matrix

$$A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}.$$

(g) Show that λ is a characteristic root of the matrix A then $K + \lambda$ is a characteristic root of the matrix $A + KI$.

(h) Solve the equation $x^3 - 3x^2 + 4 = 0$, given that two of its roots being equal.

(i) Prove that the characteristic vectors corresponding to distinct characteristic roots of a matrix are linearly independent.

(j) Find the determinant of the matrix

$$A = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$$

3. Answer ***any four*** questions from the following : 5×4=20

(a) What is linear dependence and linear independence of vectors ? Explain with *one* example.

(b) Find the characteristic roots of the matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ and verify Cayley-Hamilton theorem for this matrix.

(c) P, Q are non-singular matrices, show that if $A = \begin{bmatrix} P & O \\ O & Q \end{bmatrix}$, then

$$A^{-1} = \begin{bmatrix} P^{-1} & 0 \\ 0 & Q^{-1} \end{bmatrix}.$$

(d) Show that if λ is an eigenvalue of an orthogonal matrix, then $\frac{1}{\lambda}$ is also its eigenvalue.

(e) Find the rank of the matrix

$$\begin{bmatrix} x & 1 & 0 \\ 0 & x & 0 \\ 0 & 0 & x+3 \end{bmatrix}$$

(f) Prove that the two matrices A, $P^{-1}AP$ have the same characteristic roots.

(g) Solve the equation for x

$$\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$$

(h) Solve the following equations by Cramer's rule :

$$2x - y + 3z = 9$$
$$x + y + z = 6$$
$$x - y + z = 2$$

4. Answer ***any two*** questions from the following : 10×2=20

(a) If a, b, c are all different, solve the system of equations :

$$x + y + z = 1$$
$$ax + by + cz = k$$
$$a^2x + b^2y + c^2z = k^2$$

(b) Using the characteristic equation of

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \text{ find } A^{-1}.$$

(c) State and prove the Cayley-Hamilton Theorem.

(d) Determine non-singular matrices P and Q such that PAQ is in normal form

$$\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} \text{ where } A = \begin{bmatrix} 3 & 2 & -1 & 5 \\ 5 & 1 & 4 & -2 \\ 1 & -4 & 11 & -19 \end{bmatrix}.$$

(e) Show that if Δ is a determinant of order n, then $\Delta' = \Delta^{n-1}$.

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