Total number of printed pages–7

1 (Sem–4/FYUGP) PHY 42 MJ

2026

PHYSICS

(Major)

Paper : PHY4400204MJ

(Quantum Mechanics–I)

Full Marks : 45

Time : 2 hours

The figures in the margin indicate full marks for the questions.

1. Give very short answers the following : $1 \times 5 = 5$

(a) Write down the dimension of Planck's constant.

(b) Name an experiment that suggests wave nature of radiation.

(c) Write down Schrödinger equation for a free particle.

(d) What is Hilbert space ?

(e) What is a quantum dot ?

2. Answer the following questions : ***(any five)*** $2 \times 5 = 10$

(a) Write down *any one* limitation of classical mechanics in explaining photoelectric effect.

(b) Find the ratio of the de-Broglie wavelength to the Compton wavelength of a particle.

(c) Is it possible to simultaneously specify the kinetic energy and linear momentum of a mechanical system ? Justify your answer.

(d) A nucleon is confined to a nucleus of radius $5 \times 10^{-15} m$. Calculate the minimum uncertainty in the momentum of the nucleon.

(e) State the physical significance of the wave function $\psi(x, t)$?

(f) What is a wave packet ? How it affect the localization of the particle ?

(g) Distinguish between phase velocity and group velocity.

(h) Justify that tunnelling is a quantum mechanical phenomenon.

(i) Show that eigenvalues corresponding to Hermitian operators are real.

(j) Distinguish between a classical and a quantum harmonic oscillator.

3. Answer ***any four*** questions : 5×4=20

(a) Calculate the minimum energy required by a photon to transfer half of its energy to an electron at rest.

(b) Find the condition for two wave functions to be orthonormal. Find the probability current density for the wave function

$$\psi = \frac{e^{ikr}}{r}$$ 2½+2½=5

(c) What do you mean by expectation value of a dynamical variable? Find the expectation value of momentum for a particle described by the wavefunction

$$\psi(x) = \sqrt{\frac{2}{L}}\, sin\left(\frac{\pi x}{L}\right) \text{ for } 0 < |x| < L$$

= 0 elsewhere 2+3=5

(d) Show that if $\psi_1(\vec{r})$ and $\psi_2(\vec{r})$ are two independent solutions of the Schrödinger equation then $\psi(\vec{r}) = a_1\psi_1(\vec{r}) + a_2\psi_2(\vec{r})$ is also a solution of Schrödinger equation.

(e) Write the significance of energy time uncertainty principle. Discuss how we can estimate the range of interaction for fundamental force using energy time uncertainty relation and virtual particles.

(f) Explain *two* slit experiments with electron and discuss its significance.

(g) Discuss the properties of a Gaussian wave packet.

(h) Write down the radial wave function for 1s state of hydrogen atom. Also, compare the probabilities of a 1s electron in the hydrogen atom being at a distance a_0 from the nucleus than at a distance $a_0/2$. 1+4=5

4. Answer the following questions : ***(any one)*** $10\times1=10$

(a) What was the main discrepancy of Rayleigh-Jeans formula for black body radiation ? Derive Planck's law and discuss how the discrepancy was solved. $2+6+2=10$

(b) A particle is in an infinite potential well

$$v(x)=\begin{cases}0, & 0<x<a\\ \infty, & \text{elsewhere}\end{cases}$$

Find out the energy eigenvalues and eigenfunctions. Also schematically show the wave functions and probability densities for first three states. $8+2=10$

(c) Show that average motion of a wave packet follows Newton's second law of motion. What is the significance of Ehrenfest's theorem.

(d) (i) Explain the significance of the quantum numbers n, l and m. Can the principal quantum number of electron in hydrogen atom be zero. $3+1=4$

(ii) Show that the most probable distance of the electron from the nucleus in the ground state of hydrogen atom is equal to the Bohr's radius a_0. 6