*Total number of printed pages–7*

**1 (Sem–4/FYUGP) PHY 44 MJ**

**2026**

**PHYSICS**

( Major )

Paper : PHY4400404MJ

**( Mathematical Physics )**

*Full Marks : 45*

Time : 2 hours

*The figures in the margin indicate full marks for the questions.*

1. Answer the following questions : 1×5=5

(a) Write the two dimensional wave equation of a vibrating membrane.

(b) Find the period of the function $f(x) = 2\sin x + 4\cos x$.

(c) State Residue theorem.

(d) What do you mean by contraction of tensors?

(e) Write the expression for mean and variance of Poisson distribution.

2. Answer ***any five*** of the following questions : 2×5=10

(a) Write the orthogonality condition of sine and cosine functions.

(b) Check whether the complex function $f(z) = |z|$ is analytic or not.

(c) Evaluate $\int_C (z - z^2)\, dz$, where $C$ is upper half of the circle $|z| = 1$.



(d) State the Dirichlet conditions for the existence of a Fourier series.

(e) Write the law of transformation for the tensor

(i) $A^{p}_{qr}$

(ii) $B^{mn}_{ijk}$

(f) Prove that the contraction of the tensor $A^{p}_{q}$ is a scalar or invariant.

(g) What is singularity of an analytic function? Find the singularity of the function $f(z) = \dfrac{z+3}{(z-1)(z-2)}$ and write the type of the singularity of this function.

(h) Expand the function $f(x) = x$ for $0 < x < 2\pi$ in Fourier series.

(i) Express Laplace's equation in two dimensional Cartesian form and write its solution for positive separation constant.

(j) A coin is tossed thrice. A success is getting 1 or 6 on a toss. Find the mean and variance of the number of success.

3. Answer ***any four*** of the following questions : 5×4=20

(a) Solve the wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$

under the following conditions :

$$y(0, t) = 0,\ y(l, t) = 0,\ y(x, 0) = a \sin \frac{\pi x}{l}$$

and $\frac{\partial y(x, 0)}{\partial t} = 0$.

(b) A function is defined as

$$f(x) = \begin{cases} 0 & \text{for } -\pi < x < 0 \\ h & \text{for } 0 \le x < \pi \end{cases}$$

is termed as a square-wave function. Obtain the Fourier series expansion.

(c) State and prove the Cauchy's integral formula for complex integration.



(d) Find the value of the integral

$$\int_0^{l+i} \left(x - y - ix^2\right) dz$$ along real axis from

$z = 0$ to $z = 1$ and then along a line parallel to imaginary axis from $z = 1$ to $z = 1 + i$.

(e) What is Kronecker delta? Show that Kronecker delta is a mixed tensor of rank 2. 2+3=5

(f) Using residue theorem evaluate the integral $\int_C \frac{4 - 3z}{z(z-1)(z-2)} dz$ where $C$

is the circle $|z| = \frac{3}{2}$.

(g) Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y} + u$, where

$u(x, 0) = 3e^{-4x}$.

(h) Express the Fourier series in complex form.



Contd.

4. Answer ***any one*** of the following questions : $10\times1=10$

(a) Find the Fourier series expansion of

$$f(x)=\begin{cases}-\pi, & \text{for } -\pi<x<0\\ x, & \text{for } 0<x<\pi\end{cases}$$

Also show that $\dfrac{\pi^2}{8}=\dfrac{1}{1^2}+\dfrac{1}{3^2}+\dfrac{1}{5^2}+\ldots.$

$7+3=10$

(b) (i) Expand the following function in Taylor series :

$$f(z)=\frac{1}{z+1},\ \text{about } z=1$$ 5

(ii) Show that $\displaystyle\int_0^{\infty}\frac{dx}{1+x^2}=\frac{\pi}{2}$

(using residue theorem) 5

(c) (i) What are symmetric and antisymmetric tensors ? Prove that any second rank tensor can be written as a sum of symmetric and antisymmetric tensor. $2+4=6$

(ii) What is Levi-Civita tensor ? Prove that $\delta_{ij}\,\varepsilon_{ijk}=0$. $2+2=4$



(d) (i) What is analytic function ? Deduce Cauchy-Riemann equation as a necessary condition for a complex function to be analytic. $1+6=7$

(ii) Expand $f(z)=\dfrac{e^z}{(z-1)^2}$ in Laurent's series about $z=1$. 3

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