1 (Sem–4/FYUGP) PHY 41 MJ

2026

PHYSICS

(Major)

Paper : PHY4400104MJ

(Classical Mechanics)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1 \times 8 = 8$

(a) The number of generalised coordinates necessary to describe a diatomic molecule is

(i) 2

(ii) 3

(iii) 5

(iv) 6 *(Choose the correct answer)*

(b) For a system of N particles the transformation equations for the position vector of particles are

$\vec{r}_i = \vec{r}_i\,(q_1, q_2, ..., q_n, t)$ where

$q_1, q_2, ..., q_n$ are generalised coordinates. What is the expression for velocities?

(c) If for a system the generalised coordinate is angle (θ). Which of the following would represent the generalised force?

(i) Force

(ii) Energy

(iii) Torque

(iv) Angular momentum

(d) Lagrangian of a free particle of mass m moving in x-direction in $L = \frac{1}{2} m\dot{x}^2$.

What is it Hamiltonian?

(e) Hamilton canonical equations for a conservative system are

(i) $-\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}, -\frac{\partial p_i}{dt} = \frac{\partial H}{\partial q_i}$

(ii) $\frac{dp_i}{dt} = \frac{\partial H}{\partial p_i}, \frac{dp_i}{dt} = \frac{\partial H}{\partial q_i}$

(iii) $-\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}, \frac{dp_i}{dt} = \frac{\partial H}{\partial q_i}$

(iv) $\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}, -\frac{dp_i}{dt} = \frac{\partial H}{\partial q_i}$

(Choose the correct answer)

(f) If $v = 0{\cdot}6c$ find the x' coordinate in S'-frame of an event in S-frame at $x = 4m,\ y = 0,\ z = 0,\ t = 0$.

Contd.

(g) If the rest mass of a particle is zero, which of the following is correct?

(i) The particle is at rest for all time

(ii) The momentum of the particle is zero

(iii) The particle moves with velocity of light

(iv) None of the above

(h) A particle of mass m is moving in the potential $V(x) = -\frac{1}{2}ax^2 + \frac{1}{4}bx^4$, $(a, b)0)$. Which of the following is the position of stable equilibrium?

(i) $x = 0$

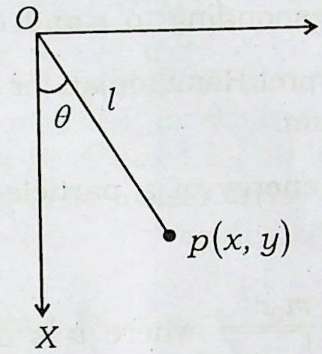
(ii) $x = \sqrt{\frac{a}{b}}$

(iii) $x = -\sqrt{\frac{a}{b}}$

(iv) None of the above

2. Answer ***any six*** questions : $2\times6=12$

(a) A simple pendulum of length l moving in xy-plane is shown in the figure. Write down the equations of constraints in the case of the pendulum.



(b) What are the two types of difficulties introduced by constraints in the solution of mechanical problem?

(c) The Lagrangian of a system in terms of generalised coordinate x is given by $L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$, where m and k are constants. Find the Euler-Lagrange equation.

(d) The Lagrangian for a particle moving in central potential on $V(r)$ is

$$L = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - V(r)$$

which coordinate is cyclic here and what is the corresponding momentum that is conserved? $1+1=2$

Contd.

(e) If the Lagrangian is given by

$$L = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2 + \dot{z}^2\right) - V(r, \theta, z)$$

Find generalised momenta corresponding to r and θ.

(f) Interpret Hamiltonian for a conservative system.

(g) The energy of a particle of rest mass m_0 is

$$E = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$ where v is the velocity of the particle. What is the kinetic energy of the particle?

Show that for $v << c$, kinetic energy is equal to $\frac{1}{2}m_0 v^2$. ½+1½=2

(h) A beam of particle of half life $2 \times 10^6 sec$ travels at a speed $0{\cdot}96C$. How much distance the beam travel before the flux falls to 50% of the initial flux?

(Half life is the time taken to reduce flux to half its initial value)

(i) If $V(x)$ is the potential energy of a particle moving in x-direction, write down the condition of stable equilibrium in terms of

$$\frac{dV(x)}{dx} \text{ and } \frac{d^2V(x)}{dx^2}.$$

(j) The mass of an electron is $9{\cdot}1 \times 10^{-31} kg$. Calculate the energy equivalent to this mass in MeV.
$[1e = 1{\cdot}6 \times 10^{-19}\text{coulomb}]$

3. Answer ***any four*** questions : 5×4=20

(a) What are holonomic and nonholonomic constraints? A system of N particles has K number of holonomic constraints and K' number of nonholonomic constraints. What is the number of degrees of freedom for the system? How many generalised coordinates are necessary to describe it? Give an example of either holonomic constraints or nonholonomic constraints. 2+1+1+1=5

(b) Use Euler-Lagrange equation to find the equation of motion of a compound pendulum in a vertical plane about a fixed horizontal axis.

(c) Lagrangian of a particle of mass m in spherical polar coordinate system with potential $V(\pi, \theta, \varphi)$ is given by

$$L = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2 + r^2 \sin^2 \theta\dot{\varphi}^2\right) - V(r, \theta, \varphi)$$

obtain the Hamiltonian and Hamilton's canonical equation. 2+3=5

(d) Define generalised momentum. What is cyclic coordinate? Show that generalised momentum corresponding to a cyclic coordinate is a constant of motion. 1+1+3=5

(e) Given the Lagrangian

$$L = \frac{1}{2}M\dot{q}_1^2 + 2M\dot{q}_2^2 - K\left(\frac{5}{4}q_1^2 + 2q_2^2 - 2q_1q_2\right)$$

where M and K are positive constants. Calculate the potential energy matrix and kinetic energy matrix. 2½+2½=5

(f) In the laboratory one particle A has velocity $V_A = +2\cdot0\times10^8\, m/sec$, another particle B has velocity $V_B = -2\cdot0\times10^8\, m/sec$. Calculate the velocity of A relative to B.
If the particle A is a photon, calculate the velocity of the photon relative to B. Is the result consistent with special relativity? 2+2+1=5

(g) Energy E and momentum P of a particle of rest mass m_0 is given by $E = \dfrac{m_0c^2}{\sqrt{1-\dfrac{v^2}{c^2}}}$

and $P = \dfrac{m_0v}{\sqrt{1-\dfrac{v^2}{c^2}}}$ respectively. Find a relation between E, P and m_0. If K is the kinetic energy of the particle show that $P = \sqrt{K^2/C^2 + 2m_0K}$. 2½+2½=5

(h) A body of rest mass m_0 and velocity $v = 0\cdot6c$ collides with another similar body at rest. After the collision the two bodies coalesce to form a composite body. Calculate the rest mass and velocity of the composite body so formed. 2½+2½=5

4. Answer ***any two*** questions : 10×2=20

(a) State D'Alembert's principle and obtain Lagrange's equation for a holonomic conservative system. What is Lagrangian? 2+7+1=10

(b) Set up the Lagrangian and the Hamiltonian for a simple pendulum. Use Hamilton's canonical equations to obtain an equation describing its motion. 2+2+6=10

(c) Obtain the formula describing variation of mass with velocity. What is rest mass ? 8+2=10

(d) A light pulse is emitted at the origin of a frame of reference S' at time $t'=0$. Its distance from the origin after time t' is given by $x'^2 = c^2 t'^2$. Use Lorentz transformation to transform this equation to an equation in x and t and show that $x^2 = c^2 t^2$.

Discuss the implications of this result.

Show that if l_0^3 is the volume of a cube at rest then $l_0^3 \sqrt{1 - v^2/c^2}$ is the volume viewed from a reference frame moving in a direction parallel to an edge of the cube with velocity v. 5+2+3=10

(e) Describe stable equilibrium and unstable equilibrium of a system with the help of potential energy curve.

Lagrangian of a system with two degrees of freedom is

$$L = \frac{m}{2}\left(\dot{x}^2 + \dot{y}^2\right) - \frac{1}{2K}\left(x^2 + y^2\right) + \alpha xy$$

where $\alpha > 0$.

Determine the eigenfrequencies of the system. 5+5=10
