2026

MATHEMATICS

(Major)

Paper : MAT0600104

(Linear Algebra)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions.

1. Answer the following as directed : 1×8=8

(a) What is the rank of

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}?$$

(b) The set consisting of only the zero vector in a vector space V is a ____ of V.

(Fill in the blank)

(c) Find $\| (0, 5) \|$.

(d) Is the set $W = \{(x, y) \in \mathbb{R}^2 : x + y = 0\}$ a subspace of $\mathbb{R}^2$?

(e) Does the set $\{(1, 0), (0, 1), (1, 1)\}$ form a basis for $\mathbb{R}^2$?

(f) If a 3×3 matrix has a rank of 2, what is the dimension of its null space?

(g) Is the map $T : V_3 \to V_3$ defined by $T(x, y, z) = (x, y)$ linear?

(h) Every constant map from one vector space to another is both one-one and onto.

(State True or False)

2. Answer any *six* questions : 2×6=12

(a) Test whether (1, 2, 3) belongs to the null space of

$$P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

(b) Find the orthogonal projection of (2, 0) on (1, 0).

(c) Write the characteristic polynomial of

$$A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$$

(d) Explain the relationship between basis and dimension.

(e) Compute the inner product of $u = (1, 2, 3)$ and $v = (2, 0, -1)$.

(f) Find the dimension of span $\{(1, 0, 0), (0, 1, 0), (1, 1, 0)\}$.

(g) Find the orthogonal projection of $y = (2, 3)$ onto the subspace spanned by $u = (1, 1,)$.

(h) In $\mathbb{R}^3$, find an orthogonal basis for the subspace spanned by (1, 1, 0), (1, 0, 1).

(i) Prove that the intersection of two subspaces of a vector space is a subspace. Give an example in $\mathbb{R}^2$ to show that the union of two subspaces is not in general a subspace.

(j) Let

$$A = \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix}$$

An eigenvalue of A is 2. Find a basis for the corresponding eigenspace.

3. Answer any *four* questions : 5×4=20

(a) Find a basis for the null space of

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & -2 & 4 \end{bmatrix}$$

(b) Show that the set {(1, 0, 1), (2, 1, 3), (0, 1, 1)} is linearly independent.

(c) Find the eigenvalues and eigenvectors of

$$B = \begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix}$$

(d) Determine whether the matrix

$$A = \begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix}$$

is diagonalizable. Justify.

(e) Apply the Gram-Schmidt process to (1, 1, 0) and (1, 0, 1).

(f) Verify whether $< u, v > = u_1 v_1 + u_2 v_2$ defines an inner product on $\mathbb{R}^2$.

(g) If $v_1, v_2, \cdots, v_r$ are eigenvectors that correspond to distinct eigenvalues $\lambda_1, \lambda_2, \cdots, \lambda_r$ of an $n \times n$ matrix A, then prove that the set $\{v_1, v_2, \cdots, v_r\}$ is linearly independent.

(h) Diagonalize the following matrix :

$$\begin{bmatrix} -6 & 4 & 0 & 9 \\ -3 & 0 & 1 & 6 \\ -1 & -2 & 1 & 0 \\ -4 & 4 & 0 & 7 \end{bmatrix}$$

4. Answer any *two* questions : 10×2=20

(a) State Cayley-Hamilton theorem. Verify the theorem for the matrix

$$C = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$$

Use the theorem to find C^{-1}. 2+4+4=10

(b) Define rank and nullity of a matrix. State and prove Rank theorem. 1+1+2+6=10

(c) Define inner product space. Show that the following is an inner product in $\mathbb{R}^2$:

$$< u, v > = 4u_1v_1 + 5u_2v_2$$

where

$$u = (u_1, u_2),\ v = (v_1, v_2) \in \mathbb{R}^2$$

Also, show that in any inner product space V

$$|< u, v >| \le ||u||.||v||,\ \forall u, v \in V$$ 2+4+4=10

(d) Given

$$A = \begin{bmatrix} 2 & -1 & 3 & 1 & -2 \\ 1 & 0 & 2 & -1 & 1 \\ 3 & -1 & 5 & 0 & -1 \end{bmatrix}$$

(i) Determine a spanning set for the null space of A.

(ii) Find the dimension of the null space.

(iii) Is the spanning set linearly independent? Justify your answer.

4+3+3=10

(e) The vectors $v_1 = (1, 2, 2)$, $v_2 = (2, 0, 1)$ are given.

(i) Apply the Gram-Schmidt process to the vectors v_1 and v_2.

(ii) Find an orthogonal basis for the subspace spanned by v_1 and v_2.

(iii) Normalize the vectors to obtain an orthonormal basis. 4+3+3=10

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