2026

MATHEMATICS

(Major)

Paper : MAT0600204

(Partial Differential Equations)

Full Marks : 45

Time : 2 hours

The figures in the margin indicate full marks for the questions.

1. (a) Under what condition the partial differential equation

$$a(x, y)u_x + b(x, y)u_y + c(x, y)u = d(x, y)$$

is said to be homogeneous? 1

(b) Classify the partial differential equation

$$u_{xx} + u_{xy} + u_{yy} + u_x = 0$$ 1

(c) Find the complete integral of

$$z = px + qy + f(p, q)$$ 1

(d) If u is the complementary function and z_1 is a particular integral of a linear partial differential equation, then what will be the general solution of this equation? 1

(e) What will be the Charpit's auxiliary equations for a partial differential equation $f(x, y, z, p, q) = 0$? 1

2. Answer any *five* parts : 2×5=10

(a) Construct a partial differential equation by eliminating a and b from

$$z = ax + by + a^2 + b^2$$

(b) Define a partial differential equation and its order.

(c) What is meant by linear and semi-linear partial differential equation?

(d) Show that the family of spheres $x^2 + y^2 + (z-c)^2 = r^2$ satisfies the first-order linear partial differential equation.

(e) Find the general solution of the first-order linear partial differential equation $xu_x + yu_y = u$.

(f) Find a complete integral of $q = 3p^2$.

(g) Find a complete integral of $(p+q)(z - xp - yq) = 1$.

(h) Reduce the equation $u_x - u_y = u$ to canonical form.

(i) Solve $a(p+q) = z$.

(j) Find a complete integral of $z = pq$.

3. Answer any *four* parts : 5×4=20

(a) Solve $z(xp - yq) = y^2 - x^2$.

(b) Eliminate the arbitrary function f from $f(x^2 + y^2 + z^2, z^2 - 2xy) = 0$ to form a partial differential equation.

(c) Find the general integrals of the linear partial differential equation

$$y^2 p - xyq = x(z - 2y)$$

(d) Find the general integral of $xzp + yzq = xy$.

(e) Reduce the equation $yu_x + u_y = x$ to canonical form and obtain the general solution.

(f) Solve the initial value problem $u_x + 2u_y = 0$, $u(0, y) = 4e^{-2y}$ by separation of variables.

(g) Find the solution of the equation

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$$

(h) Solve the equation $r + s - 2t = e^{x+y}$ where $r = \frac{\partial^2 z}{\partial x^2}$, $s = \frac{\partial^2 z}{\partial x \partial y}$, $t = \frac{\partial^2 z}{\partial y^2}$.

4. Answer any *one* part : 10

(a) Find complete integrals of the equations using Charpit's method : 5+5=10

(i) $p^2 x + q^2 y = z$

(ii) $pq = px + qy$

where $p = \dfrac{\partial z}{\partial x}$, $q = \dfrac{\partial z}{\partial y}$

(b) Determine the general solution of $4u_{xx} + 5u_{xy} + u_{yy} + u_x + u_y = 2$.

(c) Find the integral surface of the linear partial differential equation

$$x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$$

which contains the straight line $x + y = 0$, $z = 1$.

(d) Find the complete integral of

$$p_1^3 + p_2^2 + p_3 = 1$$

using Jacobi's method.

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