*Total number of printed pages–7*

**1 (Sem–4/FYUGP) MAT 41 MJ**

**2026**

**MATHEMATICS**

( Major )

Paper : MAT4400104MJ

**( Real Analysis )**

*Full Marks : 60*

Time : 2½ hours

*The figures in the margin indicate full marks for the questions.*

1. Answer the following questions : $1\times8=8$

(a) Determine the set

$A = \{x \in \mathbb{R} : |2x+3| < 7\}$.

(b) Write the completeness property of real numbers.

(c) If $S = \{1/n + 1 : n \in \mathbb{N}\}$, then find *inf* $S$.

(d) Define the range of a sequence of real numbers. Also, give *one* example.

(e) What is a bounded sequence ? Give *one* example.

(f) Find $lim\left(\frac{3n+5}{n}\right)$.

(g) State Cauchy's criterion for convergence of a series $\sum_n X_n$.

(h) The positive term series $\sum \frac{1}{n^p}$ is convergent if and only if

(i) $p > 0$



(ii) $p > 1$

(iii) $0 < p < 1$

(iv) $p \leq 1$ (*Write the correct option*)

2. Answer ***any six*** of the following questions : $2\times6=12$

(a) If $a \in \mathbb{R}$ and $a \neq 0$, then show that $a^2 > 0$. Hence show that $1 > 0$.

(b) If $z, a \in \mathbb{R}$ with $z + a = a$, then prove that $z = 0$.

(c) Show that for all $a, b \in \mathbb{R}$, $|ab| = |a||b|$.

(d) Define $\varepsilon$-neighbourhood of $a \in \mathbb{R}$. Is it true that if $x$, $y$ belong to the $\varepsilon$-neighbourhoods of $a$, $b$, respectively, then $x+y$ belong to the $\varepsilon$-neighbourhood of $a+b$?

(e) Show that $lim\left(\frac{\sin n}{n}\right) = 0$.

(f) Let $(x_n)$ be a sequence of real numbers defined by $x_1 = 2,\ x_{n+1} = 2 + {}^{1}\!/\!{}_{x_n}\ ,\ n \in \mathbb{N}$.

If $x = lim(x_n)$, find $x$.

(g) Show that the sequence $\left({}^{1}\!/\!{}_{n}\right)$ is a Cauchy sequence.

(h) Examine the convergence or divergence of the sequence $\left((-1)^n\right)$.

(i) Define alternating series. Give *one* example.

(j) Examine the convergence of the series

$$\sum_{n=1}^{\infty} \frac{1}{n^2 - n + 1}.$$

3. Answer ***any four*** of the following questions : 5×4=20

(a) State and prove the Archimedean property of real numbers.



(b) Show that between any two real numbers $x$ and $y$ with $x < y$, there exists a rational number $r$ such that $x < r < y$.

(c) (i) Show that no smallest positive real number can exists. 2

(ii) Solve the following inequality :

$$|2x - 1| \leq x + 1.$$ 3

(d) Show that the set $\mathbb{R}$ of of real numbers is not countable.

(e) State and prove the squeeze theorem of real sequence.

(f) Show that if $c > 0$, then $lim\left(c^{1/n}\right) = 1$.

(g) Examine the convergence of the series

$$\sum_{n=1}^{\infty} \frac{n}{n^2 + 1}.$$

(h) If $X = (x_n)$ is a decreasing sequence with $lim\ x_n = 0$, and if the partial sums $(s_n)$ of $\Sigma y_n$ are bounded, then prove that the series $\Sigma x_n y_n$ is convergent.

4. Answer ***any two*** of the following questions : $10\times2=20$

*(a)* Prove that a sequence of real numbers is convergent if and only if it is a Cauchy sequence.

*(b)* State and prove monotone convergence theorem of real numbers.

*(c)* If $X=(x_n)$ and $Y=(y_n)$ are sequences of real numbers that converged to $x$ and $y$, respectively, then prove that the sequences $X+Y$ and $X.Y$ converge to $x+y$ and $xy$ respectively.

*(d)* Prove that if a series $\sum_n x_n$ is absolutely convergent, then any rearrangement $\sum_k y_k$ of $\sum_n x_n$ is also convergent to the same value.

*(e)* *(i)* Show that the geometric series $\sum_{n=0}^{\infty} r^n$ converges to $\frac{1}{1-r}$ for $|r|<1$. 3



*(ii)* Examine the convergence of the $p$-series $\sum_n \frac{1}{n^P}$ for $p \geq 1$. 7

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