1 (Sem–4/FYUGP) MAT 42 MJ

2026

MATHEMATICS

(Major)

Paper : MAT4400204MJ

(Complex Analysis–I)

Full Marks : 45

Time : 2 hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1\times5=5$

(a) Find $\lim\limits_{z\to i} \frac{iz^3-1}{z+1}$.

(b) A composition of continuous function is

(i) pointwise continuous

(ii) discontinuous

(iii) itself continuous

(iv) None of the above

(Choose the correct answer)

(c) The value of $Log(-ei)$ is

(i) $\pi/2-i$

(ii) i

(iii) $1+i$

(iv) $1-\frac{\pi}{2}i$

(Choose the correct answer)

(d) The power expression of $\cos z$ is

(i) $\frac{e^{iz}+e^{-iz}}{2}$

(ii) $\frac{e^{z}+e^{-z}}{2}$

(iii) $\frac{e^{iz}+e^{-iz}}{2i}$

(iv) None of the above

(Choose the correct answer)

(e) What is Jordan arc?

2. Answer **any five** of the following questions : $2\times5=10$

(a) If $f(z)=\frac{z^2}{z}\,(z\neq0)$, where $z=x+iy$, then write $f(z)$ in the form $f(z)=u(x,y)+iv(x,y)$.

(b) Show that : $\lim\limits_{z\to\infty}\frac{2z^3-1}{z^2+1}=\infty$

(c) If $f(z)=z-\bar{z}$; then show that $f'(z)$ does not exist at any point.

(d) Find the singular point of
$$f(z)=\frac{z^2+1}{(z+2)(z^2+2z+2)}.$$

(e) If $e^z = 1+\sqrt{3}i$ then find z in form of $z = x+iy$.

(f) Find the principal value of $(-i)^i$.

(g) Evaluate : $\int_1^2 \left(\frac{1}{t}-i\right)^2 dt$

(h) Evaluate : $\int_C \frac{exp(2z)}{z^4} dz$, where 'C' is the positively oriented unit circle $|z|=1$.

(i) If $f(z)=\frac{z}{\bar{z}}$, find $\lim_{z\to 0} f(z)$, if it exists.

(j) Write the function $f(z)=z+\frac{1}{z}\,(z\neq 0)$ in the form $f(z)=u(r,\theta)+iv(r,\theta)$.

3. Answer **any four** of the following questions :

5×4=20

(i) Sketch the set $|z-2+i|\leq 1$ and determine its domain.

(ii) Show that the three cubic roots of $-8i$ lie at the vertices of an equilateral triangle that is inscribed in a circle of radius 2 centred at the origin.

(iii) If z_1 and z_2 are complex numbers then show that

$\sin(z_1+z_2)=\sin z_1 \cos z_2 + \cos z_1 \sin z_2$.

(iv) Suppose that a function $f(z)$ is analytic at a point $z_0 = z(t_0)$ on a differentiable arc $z=z(t)\,(a\leq t\leq b)$. Show that if $w(t)=f(z(t))$ then $w'(t)=f'(z(t))\,z'(t)$ when $t=t_0$.

(v) Let 'C' be the arc of the circle $|z|=2$ from $z=2$ to $z=2i$, that lies in the 1st quadrant, then show that

$$\left|\int_C \frac{z-2}{z^2+1}\,dz\right| \leq \frac{4\pi}{15}.$$

(vi) Using anti-derivative, evaluate the integral $\int_C z^{1/2}\,dz$, where 'C' is a contour from $z=-3$, to $z=3$ that except for its end points, lies above the x-axis.

(vii) Show that $\exp(2 \pm 3\pi i) = -e^2$.

(viii) If $f(z) = \dfrac{z+2}{z}$ and C is the semi circle $z = 2e^{i\theta}, (0 \le \theta \le \pi)$, then evaluate $\int\limits_C f(z)\, dz$.

4. Answer ***any one*** of the following questions : 10

(a) State and prove Cauchy-Riemann equations of an analytic function in polar form.

(b) (i) Let u and v denote the real and imaginary components of the function 'f' defined by the equations :

$$f(z) = \begin{cases} \dfrac{(\bar{z})^2}{z} & , \text{ when } z \neq 0 \\ 0 & , \text{ when } z = 0 \end{cases}$$

Verify the Cauchy-Riemann equations at the origin $z = (0, 0)$. 5

(ii) Show that if a function $f(z) = u(x, y) + iv(x, y)$ and its conjugate $\overline{f(z)}$ are both analytic in a domain D, then $f(z)$ must be constant throughout D. 5

(c) State and prove the fundamental theorem of algebra.

(d) If a function 'f' is analytic at all points interior to and on a simple contour 'C', then prove that $\int\limits_C f(z)\, dz = 0$.
