Total number of printed pages–8

1 (Sem–4/FYUGP) MAT 43 MJ

2026

MATHEMATICS

(Major)

Paper : MAT4400304MJ

(Analytical Geometry)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1 \times 8 = 8$

(a) Write down the formulae of transformation from one pair of rectangular axes to another with same origin.

(b) A second degree homogeneous equation of the form $ax^2 + 2hxy + by^2 = 0$ represents ________ through the origin. *(Fill in the blank)*

(c) Under what condition the comic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ may represent a parabola?

(d) Find the condition that the lines $y = mx$ and $y = m'x$ will be two conjugate diameters of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(e) Any section of a sphere by a plane is a circle. *(State True **or** False)*

(f) What is the cylindrical coordinates of the point whose Cartesian coordinates are (x, y, z)?

(g) The graph of $(x-3)^2 + (y-2)^2 + (z+1)^2 = 16$ is a ________ of radius ________ centred at ________. *(Fill in the blanks)*

(h) In 3-space, the vector from $A(0, -2, 5)$ to $B(3, 4, -1)$ is $\overrightarrow{AB} = ?$

2. Answer **any six** questions : $2 \times 6 = 12$

(a) Find the equation of $\frac{x}{a} + \frac{y}{b} = 2$ when the origin is shifted to (a, b).

(b) If $ax + by$ transforms to $a'x' + b'y'$ under rotation of axes, then show that $a^2 + b^2 = a'^2 + b'^2$.

(c) Show that $7x^2 - 48xy - 7y^2 - 20x + 140y + 300 = 0$ represents a rectangular hyperbola.

(d) Define pole and polar of a conic.

(e) Find the nature of the following conic $\frac{5}{r} = 2 - 2\cos\theta$.

(f) What do you mean by quadric surface?

(g) Find the equation of the sphere whose centre is $(2, 3, -4)$ and radius is 5.

(h) Find the unit vector that has the same direction as $\vec{V} = 2\hat{i} + 2\hat{j} - \hat{k}$.

(i) Find the vector of length 2 that makes an angle of $\frac{\pi}{4}$ with the positive x-axis.

(j) Let $\vec{u} = \hat{i} - 5\hat{k}, \vec{v} = 2\hat{i} - 4\hat{j} + \hat{k}$ and $\vec{w} = 3\hat{i} - 2\hat{j} + 5\hat{k}$ then $\vec{U} \cdot (\vec{V} \times \vec{W}) = ?$

3. Answer ***any four*** questions : 5×4=20

(a) Prove that $g^2 + f^2$ obtained from $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ remain invariant under transformation of rotation.

(b) Find the product of the perpendiculars from (x_1, y_1) to the lines represented by $ax^2 + 2hxy + by^2 = 0$.

(c) Show that three normals can be drawn to a parabola from a given point and the sum of the ordinates of the feet of the normals is zero.

(d) Prove that the equations of asymptotes to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are $y = \pm\frac{b}{a}x$.

(e) A sphere of radius k passes through the origin and meets the axes in A, B and C. Prove that the locus of the centroid of the triangle ABC is the sphere $9(x^2 + y^2 + z^2) = 4k^2$.

(f) Find the surfaces generated by the equations

(i) $r = a$ (constant), (ii) $\theta = \alpha$ (constant)

(iii) $\phi = \beta$ (constant).

(g) Write the direction cosines of a nonzero vector $\vec{V} = v_1\hat{i} + v_2\bar{j} + v_3\hat{k}$. Also find the direction cosines of the vector $\vec{V} = 3\hat{i} - 2\hat{j} - 6\hat{k}$.

(h) Find parametric equations of the line L passing through the points $P_1(2, 4, -1)$ and $P_2(5, 0, 7)$.

4. Answer ***any two*** questions : 10×2=20

(a) (i) Show that the two straight lines $x^2\left(\tan^2\theta+\cos^2\theta\right)-2xy\tan\theta+y^2\sin^2\theta=0$ make angles with the x-axis such that the difference of their tangents is 2. 5

(ii) Show that the lines joining the origin to the points of intersection of the curves

$ax^2+2hxy+by^2+2gx=0$ and

$a'x^2+2h'xy+b'y^2+2g'x=0$ will be at right angle if $g(a'+b')=g'(a+b)$ 5

(b) (i) Obtain the condition under which the line $lx+my+n=0$ may touch the hyperbola $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$. 5

(ii) If the points of intersection of the ellipses $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ and $\dfrac{x^2}{\alpha^2}+\dfrac{y^2}{\beta^2}=1$ be the ends of conjugate diameters of the former, prove that $\dfrac{a^2}{\alpha^2}+\dfrac{b^2}{\beta^2}=2$. 5

(c) Obtain the equation of the chord of the conic $\dfrac{l}{r}=1+e\cos\theta$, joining the two points on the conic, whose vectorial angles are $(\alpha+\beta)$ and $(\alpha-\beta)$

Hence find the equation of the tangent to the conic at the point having vectorial angle x. 8+2=10

(d) (i) Find the equation of the cylinder whose guiding curve is $f(x,y)=0,\ z=0$ and generators are parallel to $\dfrac{x}{l}=\dfrac{y}{n}=\dfrac{z}{n}$. 5

(ii) The section of a cone whose guiding curve is $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1,\ z=0$ by the plane $x=0$ is a rectangular hyperbola, show that the locus of the vertex is $\dfrac{x^2}{a^2}+\dfrac{y^2+z^2}{b^2}=1$. 5

(e) (i) Find the orthogonal projection of $\vec{v}=\hat{i}+\hat{j}+\hat{k}$ on $\vec{b}=2\hat{i}+2\hat{j}$ and then find the vector component of $\vec{v}$ orthogonal to $\vec{b}$. 5

(ii) Show that the lines L_1 and L_2 intersect and find their point of intersection.

$$L_1 : x = 2 + t,\ y = 2 + 3t,\ z = 3 + t$$

$$L_2 : x = 2 + t,\ y = 3 + 4t,\ z = 4 + 2t$$

5
